An invitation to the study of Families of Complex Manifolds

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We deal with compact complex manifolds X. Already in the early days of complex geometry it was realized that it is of fundamental interest to view these as moving in families, i.e., a fibration $\pi : \mathfrak{X} \to S$ where $\pi^{-1}(s) =: X_s$ are the manifolds of interest as s moves in the parameter space S. A first important example, which will be discussed in the talk, is where X_s is defined as the 0-set of a polynomial whose coefficients depend on s, e.g., cubics varying in the (projective) plane. Another important classical example arises when one defines a family $\mathfrak{X}$ of 1-dimensional tori by X_s being the quotient of the complex plane, $X_s := \mathbb{C}/\Gamma_s$, by a lattice which is presented by generators which depend on s, e.g., $\Gamma_s = Span_{\mathbb{Z}}(1, s)$ where s runs in the upper halfplane.

It is possible, as in the case of both examples mentioned above, that there are natural equivalence relations defined on S. For example, the complex general linear group acts on polynomials with the induced action on their 0-sets defining a notion of equivalence on S. The case of tori seems more subtle, because the natural equivalence takes place over the integers, being defined by the action of integral matrices on the generators of the lattice.

Various ideas for constructing moduli (parameter) spaces S will be discussed. In certain contexts there are useful (period) maps of S which will be introduced. Describing the images of these maps has been of interest since the 19th century and has led to strong activity in modern complex geometry. A very rough idea of current research in this area and the type of open problems which are being considered will be given.